Solar Pressure Control of a Dual-Spin Satellite

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Nutation damping and attitude control of a spinning satellite using solar radiation pressure is studied for the general case of attutide motion in an arbitrary orbit. The nonlinear, nonautonomous, coupled equations of motion involving large numbers of parameters are only amenable to a numerical approach. The response data, presented as functions of the system parameters, show the controller's effectivenss in damping the most severe disturbances in a fraction of an orbit. The controller provides versatility to the satellite, enabling it to change its preferred orientation in orbit and, hence, undertake diverse missions. An illustrative example using mass properties representative of INTELSAT IV and the Canadian communications satellite (Anik-1) demonstrates the feasibility of the concept. The semipassive character promises an increased lifetime with an effective reduction in cost.

Nomenclature

A_i = area of the set of plates, $i = 1, 2, 3$
C_i = solar parameters, $i = 1, 2, 3$
$E(\theta)$ = $(1 + e)^3/(1 = e\cos\theta)^4$
I = inertia parameter, I_x/I_y
I_x, I_y, I_z = principal moments of inertia of the satellite
I_{xp}, I_{xs} = moments of inertia of the platform (II) and the rotor (I) about the axis of symmetry, respectively, (Fig. 2)
J = platform inertia fraction, I_{xp}/I_x
K = viscous damping parameter, $K_d R_p^{3/2}/(\mu I_y)$
K_d = viscous damping coefficient
P = pericenter
Q_i = generalized moments, $i = \gamma, \beta, \lambda$
R = distance between the satellite center of mass and the centre of force 0
R_p = distance between the pericenter and the center of force
e = eccentricity
h_α = constant of motion
i = inclination of the orbital plane with the ecliptic
$\bar{i}, \bar{j}, \bar{k}$ = unit vectors along x, y and z axes, respectively
ε_i = distance of the center of pressure of a set of plates from the satellite center of mass, $i = 1, 2, 3$ (Fig. 2)
$\bar{n}, \bar{s}$ = unit vectors along the surface normal and the direction of the reflected light, respectively
$\bar{u}$ = unitvector in the direction of the sun, $u_i\bar{i} + u_j\bar{j} + u_k\bar{k}$
u_i = $\cos\phi(\sin\gamma\cos\beta\cos\theta + \sin\beta\sin\theta) + \sin\phi\{\cos i(\sin\gamma\cos\beta\sin\theta - \sin\beta\cos\theta) - \sin i\cos\beta\cos\gamma\}$
u_j $\omega = 0$ = $\cos\phi\cos\gamma\cos\theta + \sin\phi(\cos i\cos\gamma\sin\theta + \sin i\sin\gamma)$
u_k = $\cos\phi(\sin\gamma\sin\beta\cos\theta - \cos\beta\sin\theta) + \sin\phi\{\cos i(\sin\gamma\sin\beta\sin\theta + \cos\beta\cos\theta) - \sin i\sin\beta\cos\gamma\}$
p_o = solar radiation pressure
$\bar{r}$ = position vector to plate element from the satellite c.m.
t = time
x, y, z = principal body coordinates
x', y', z' = inertial coordinates
x_0, y_0, z_0 = rotating coordinate system with x_0 normal to the orbital plane and y_0 along the local vertical
$x_1, y_1, z_1; x_2, x_2, z_2$ = intermediate body coordinates resulting from rotations γ and β about z_0 and y_1 axes, respectively
$\alpha, \beta, \gamma, \lambda$ = attitude angles
δ_i = plate rotations, $i = 1, 2, 3$
θ = orbital angle
μ = gravitational constant
μ_i, ν_i = system gains, $i = \gamma, \beta, \lambda$
$\gamma_c, \beta_c, \lambda_c$ = position control parameters
$\gamma_f, \beta_f, \lambda_f$ = final orientation
ξ = angle of incidence
ρ, τ = reflectivity and transmissibility of plate surfaces, respectively
σ = spin parameter
ϕ = solar aspect angle, angle between the direction of the sun and the line of nodes NN'

Subscripts
0 = initial condition

Superscripts
$(\dot{\ }), (\ ')$ = differentiation with respect to t and θ, respectively

Introduction

THE influence of environmental forces on the librational motion of a satellite has received some attention in recent years. The destabilizing effect of the solar radiation pressure on the planar attitude dynamics of a flat plate satellite was analyzed by Modi and Flanagan.[1] They also explored a possibility of utilizing this force to advantage through a semipassive controller.[2] The concept was further generalized by Modi and Tschann[3] through a displacement and velocity sensitive controller. The authors presented optimization plots for transient and steady-state responses and suggested a possible controller configuration. Modi and Kumar[4] studied, in the presence of the solar pressure, the librational dynamics of a more realistic cylindrical model and subsequently demonstrated the versatility of the semipassive controller in achieving librational damping and spatial attitude control in orbit.[5] The control system, however, had two main disadvantages: the inability of the controller to eliminate limit cycle oscillations in elliptical orbits, and the practical difficulty of providing sliding movements to the controller links in a hostile space environment.

The analyses discussed above pertain to gravity oriented satellites. On the other hand, utilization of solar forces for attitude control of spinning satellites remains virtually unexplored. This is unfortunate, because, in many space applications, a satellite with a directional sensor has a preferred orientation which is normally achieved by mounting the device

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Presented as Paper 72-918 at the AIAA/AAS Astrodynamics Conference, Palo Alto, Calif., September 11-12, 1972; submitted October 3, 1972; revision received January 30, 1972. This investigation was supported by the Defense Research Board of Canada under Grant 9551-18.

Index category: Spacecraft Attitude Dynamics and Control.

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on a stabilized platform aboard the spinning satellite. The spin, through a gyroscopic moment, provides stability while a platform, despun by control moments, tracks a given object in space. The concept of solar pressure control provides an exciting possibility of stabilizing the entire system through a semipassive approach.

It would be appropriate here to mention the experiment aboard Mariner IV spacecraft,[6] conducted on depletion of the attitude control gas, to align the roll axis along the sun-line using passive solar radiation control. Each of the four solar panels was provided with a rotatable solar pressure vane for this purpose. Unfortunately, one of the vanes proved to be inoperative during a major portion of the mission. However, subsequent reactivation of the vane enabled the solar pressure control system, in conjunction with active gyros, to maintain the spacecraft attitude within 1° of the sun-line.

This paper investigates the feasibility of the general three-dimensional nutation damping and attitude control of a spinning satellite using a solar controller sensitive to angular displacement and velocity. Both the circular and elliptic orbit cases are considered for an arbitrary inclination of the orbital plane with reference to the ecliptic. The nonlinear, nonautonomous, coupled equations are analyzed numerically, and the response data presented as functions of the system parameters, orbit eccentricity, and the solar aspect angle. Versatility of the controller in achieving any orientation is demonstrated. A preliminary configuration of the proposed controller is also included. An illustrative example towards the end presents results on the expected performance of Anik-1 and INTELSAT IV series of satellites.

Formulation of the Problem and Controller Configuration

Equations of motion

Figure 1 shows an axisymmetric cylindrical satellite with the c.m. S moving in a Keplerian orbit about the center of force O. The satellite consists of a central body I, spinning at a constant average angular velocity, connected to a stabilized platform

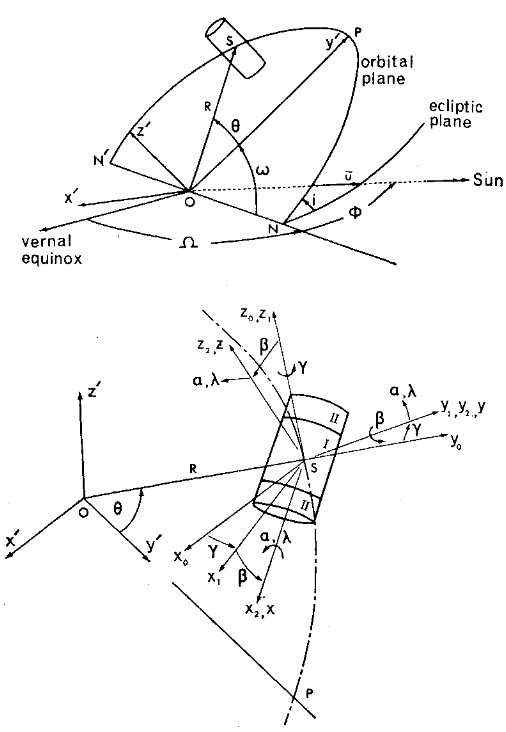

Fig. 1 Geometry of motion.

II through a viscous damper effective in axial rotation. The spatial orientation of the axis of symmetry is completely specified by two successive rotations γ and β, referred to as roll and pitch, respectively, which define the attitude of the principal axes x, y, z with respect to the inertial reference frame x', y', z'. The rotor and the platform spin in the x, y, z reference with angular velocities $\dot{\alpha}$ and $\dot{\lambda}$, respectively.

α being a cyclic coordinate, a first integral of motion defining $\dot{\alpha}$ is given by,

$$\dot{\alpha} - \dot{\gamma}\sin\beta + \dot{\theta}\cos\beta\cos\gamma = h_\alpha \tag{1}$$

Since h_α is a measure of the rotor spin, a dimensionless spin parameter, σ, defined as

$$\sigma = \dot{\alpha}/\dot{\theta}\,|_{\theta=\beta=\gamma=0} = h_\alpha/\dot{\theta}\,|_{\theta=0} - 1 \tag{2}$$

may be used to eliminate the cyclic coordinate α.

Neglecting orbital perturabtions due to attitude motion and changing the independent variable from t to θ through the use of the Keplerian orbital relations, the classical Lagrangian formulation yields the governing equations of motion in the γ, β and λ degrees of freedom as[7]:

$$\begin{aligned}\gamma'' - 2\beta'(\gamma'\tan\beta - \cos\gamma) - (\beta' - \sin\gamma)\sec\beta \times \\ [(1-J)I(\sigma+1)\{(1+e)/(1+e\cos\theta)\}^2 + \\ JI(\lambda' - \gamma'\sin\beta + \cos\beta\cos\gamma)] + \{3(I-1)/(1+e\cos\theta) - 1\} \\ \sin\gamma\cos\gamma - \{2e\sin\theta/(1+e\cos\theta)\}(\gamma' + \cos\gamma\tan\beta) = Q_\gamma\end{aligned} \tag{3a}$$

$$\begin{aligned}\beta'' - \gamma'\cos\gamma - \{2e\sin\theta/(1+e\cos\theta)\}(\beta' - \sin\gamma) + \\ (\gamma'\cos\beta + \cos\gamma\sin\beta)[(1-J)I(\sigma+1)\times \\ \{(1+e)/(1+e\cos\theta)\}^2 + JI(\lambda' - \gamma'\sin\beta + \cos\beta\cos\gamma) + \\ (\gamma'\sin\beta - \cos\beta\cos\gamma)] - 3\{(I-1)/(1+e\cos\theta)\}\times \\ \sin^2\gamma\sin\beta\cos\beta = Q_\beta\end{aligned} \tag{3b}$$

$$\begin{aligned}\lambda'' - \gamma''\sin\beta - \{2e\sin\theta/(1+e\cos\theta)\}(\lambda' - \gamma'\sin\beta + \\ \cos\beta\cos\gamma) - \beta'\gamma'\cos\beta - \gamma'\cos\beta\sin\gamma - \\ \beta'\cos\gamma\sin\beta + (K/JI)\{(1+e)^{3/2}/(1+e\cos\theta)^2\}\times \\ [\lambda' - \gamma'\sin\beta + \cos\beta\cos\gamma - (\sigma+1)\times \\ \{(1+e)/(1+e\cos\theta)\}^2] = Q_\lambda\end{aligned} \tag{3c}$$

These highly nonlinear, nonautonomous, coupled equations of motion do not possess any known closed form solution. One is, therefore, forced to resort to a numerical approach to gain some appreciation as to the system performance.

Controller Configuration

A controller, in general, consists of light, highly reflective plates (membrane) suitably mounted on the platform to be stabilized. The control moments resulting from the solar radiation force on the plates may be varied by changing any one of the following: 1) the distance between the center of pressure and the satellite center of mass by translating the plate support; 2) the area of the membrane through wrapping or unfurling portions of it; and 3) the projected area of the plate as seen by the sun by rotating the plate.

Practical considerations make the last alternative the most attractive, especially, when servomotors can be located within the controlled environment of the spacecraft. In order that the three degrees of freedom of the system, namely the roll γ, the pitch β, and platform yaw λ, be controlled independently, it is necessary to provide at least three independent plate rotations δ_1, δ_2, and δ_3.

Various controller configurations were studied which, in general, yield expressions for the generalized moments of the form:

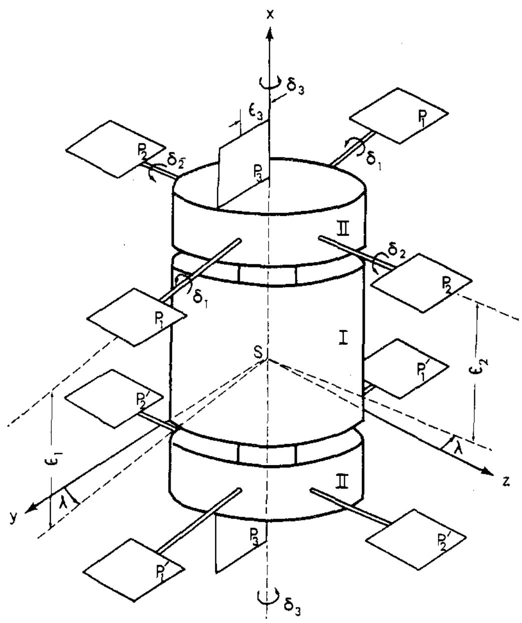

Fig. 2 Controller configuration.

$$Q_\gamma = Q_\gamma(\delta_1, \delta_2, \delta_3)$$
$$Q_\beta = Q_\beta(\delta_1, \delta_2, \delta_3)$$
$$Q_\lambda = Q_\lambda(\delta_1, \delta_2, \delta_3) \quad (4)$$

where the functions in the right hand side are transcendental.

As the rotations δ_1, δ_2, and δ_3 are real, the simultaneous maxima $|Q_\gamma|_{\max}$, $|Q_\beta|_{\max}$ and $|Q_\lambda|_{\max}$, for which the set of Eqs. (4) possesses a real solution, represent the physical limit on the generalized moments that a particular controller configuration can yield. The problem of determinating the maximum values of Q_γ, Q_β, and Q_λ, which would satisfy the above criterion is, in general, a complex one.

Figure 2 shows the schematic diagram of the proposed semi-passive controller. It consists of five sets of plates $P_i(i = 1, 2, 3)$ and $P_j'(j = 1,2)$ with their axes mounted on the platform. The plates are permitted rotations $\delta_i(i = 1, 2, 3)$ as shown in the figure. At a given instant, the set P_3, controlling the λ motion, operates in conjunction with the sets $P_i(i = 1,2)$ or $P_j'(j = 1, 2)$ or $P_iP_j'(i \neq j)$, which provide corrective torques in the γ and β degrees of freedom.

The determination of the moments due to the solar radiation pressure is somewhat involved. Figure 3 shows a plate in an arbitrary orientation with respect to the sun. The force on an elemental plate area dA is given by

$$d\bar{F} = -p_0 dA|\cos\xi|\{(1-\tau)\bar{u} + \rho\bar{s}\} \quad (5)$$

with the resulting moment about the satellite c.m. as

$$\bar{M} = \int_A \bar{r} \times d\bar{F} \quad (6)$$

Expressing the angle of incidence ξ and the unit vectors $\bar{u}$ and $\bar{s}$ as functions of the attitude angles, the solar aspect angle ϕ, the plate rotations δ_i and evaluating the integral in Eq. (6) yield the desired expression for $\bar{M}$. The application of the principle of virtual work finally leads to the generalized moments Q_i. The expressions are rather lengthy, however, ignoring the terms of the order $(1\text{-}\tau\text{-}\rho)/2\rho$ compared to unity, which is justifiable for surfaces of high reflectivity, results in a considerable simplification:

$$Q_\gamma = E(\theta)[C_1|u_i\cos\delta_1 + (u_j\sin\lambda - u_k\cos\lambda)\sin\delta_1| \times \{u_i\cos\delta_1 + (u_j\sin\lambda - u_k\cos\lambda)\sin\delta_1\}\sin\delta_1\sin\lambda + C_2|u_i\cos\delta_2 + (u_j\cos\lambda + u_k\sin\lambda)\sin\delta_2| \times \{u_i\cos\delta_2 + (u_j\cos\lambda + u_k\sin\lambda)\sin\delta_2\}\sin\delta_2\cos\lambda]\sec\beta \quad (7a)$$

$$Q_\beta = E(\theta)[C_1|u_i\cos\delta_1 + (u_j\sin\lambda - u_k\cos\lambda)\sin\delta_1| \times \{u_i\cos\delta_1 + (u_j\sin\lambda - u_k\cos\lambda)\sin\delta_1\}\sin\delta_1\cos\lambda - C_2|u_i\cos\delta_2 + (u_j\cos\lambda + u_k\sin\lambda)\sin\delta_2| \times \{u_i\cos\delta_2 + (u_j\cos\lambda + u_k\sin\lambda)\sin\delta_2\}\sin\delta_2\sin\lambda] \quad (7b)$$

$$Q_\lambda = -E(\theta)(C_3/JI)|-u_j\sin(\delta_3+\lambda) + u_k\cos(\delta_3+\lambda)| \times \{-u_j\sin(\delta_3+\lambda) + u_k\cos(\delta_3+\lambda)\} \quad (7c)$$

where the solar parameters are given by

$$C_i = \pm(2\rho p_0 R_p^3/\mu Iy)A_i\varepsilon_i,\ i = 1, 2$$
$$(+\text{ for } P_i',\ -\text{ for } P_i)$$
$$C_3 = (2\rho p_0 R_p^3/\mu Iy)A_3\varepsilon_3 \quad (8)$$

Controller Characteristics

The generalized moment Q_i are controlled in a velocity and position sensitive manner according to the relations:

$$Q_\gamma = -\mu_\gamma\gamma' - \nu_\gamma(\gamma - \gamma_c) \quad (9a)$$
$$Q_\beta = -\mu_\beta\beta' - \nu_\beta(\beta - \beta_c) \quad (9b)$$
$$Q_\lambda = -\mu_\lambda\lambda' - \nu_\lambda(\lambda - \lambda_c) \quad (9c)$$

where the system gains μ_i, ν_i are chosen according to some suitable criterion, such as, the least time of damping or the maximum permissible displacement during a nutation cycle. The position control parameters γ_c, β_c, and λ_c, however, are functions of the desired final orientation γ_f, β_f, and λ_f, and are obtained from the equilibrium considerations of the controlled system (Eq. 3). For nutation damping in a circular orbit, this results in

$$\gamma_c = \beta_c = 0,\ \lambda_c = -(1/\nu_\lambda)K\sigma/JI$$

The controller may be unable to provide the corrective moments demanded by the system at all times due to its physical limitations. It is, therefore, necessary to determine the region in the Q_γ, Q_β, Q_λ space within which the system of transcendental equations (7), possesses real solutions for the plate rotations δ_1, δ_2 and δ_3.

Q_λ being a function δ_3 only, its maximum attainable value can easily be found,

$$|Q_\lambda|_{\max} = E(\theta)(C_3/JI)(u_j^2 + u_k^2) \quad (10a)$$

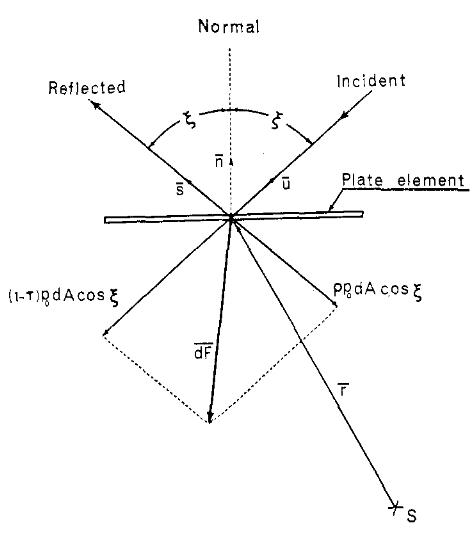

Fig. 3 Radiation force on a plate element

But Q_λ, Q_β are functions of δ_1 and δ_2, hence, it is necessary to specify a rational criterion for the controller operation. In the present analysis, based on physical considerations, it is taken to be the maximum of the function $(Q_\lambda{}^2 + Q_\beta{}^2)^{1/2}$. This results in

$$|Q_\gamma|_{\max} = |E(\theta)[C_1|u_i \cos\delta_{1m} + (u_j \sin\lambda - u_k \cos\lambda)\sin\delta_{1m}| \times \{u_i \cos\delta_{1m} + (u_j \sin\lambda - u_k \cos\lambda)\sin\delta_{1m}\}\sin\delta_{1m}\sin\lambda + C_2|u_i \cos\delta_{2m} + (u_j \cos\lambda + u_k \sin\lambda)\sin\delta_{2m}| \times \{u_i \cos\delta_{2m} + (u_j \cos\lambda + u_k \sin\lambda)\sin\delta_{2m}\}\sin\delta_{2m}\cos\lambda] \times \sec\beta| \quad (10b)$$

$$|Q_\beta|_{\max} = |E(\theta)[C_1|u_i \cos\delta_{1m} + (u_j \sin\lambda - u_k \cos\lambda) \times \sin\delta_{1m}|\{u_i \cos\delta_{1m} + (u_j \sin\lambda - u_k \cos\lambda)\sin\delta_{1m}\} \times \sin\delta_{1m}\cos\lambda - C_2|u_i \cos\delta_{2m} + (u_j \cos\lambda + u_k \sin\lambda) \times \sin\delta_{2m}|\{u_i \cos\delta_{2m} + (u_j \cos\lambda + u_k \sin\lambda) \times \sin\delta_{2m}\}\sin\delta_{2m}\sin\lambda]| \quad (10c)$$

where

$$\delta_{1m} = -\pi/2 + \{\tan^{-1}(u_i/u_k) + \sin^{-1}[u_i/3(u_i{}^2 + u_k{}^2)^{1/2}]\}/2$$

$$\delta_{2m} = \pi/2 - \{\tan^{-1}(u_i/u_j) + \sin^{-1}[u_i/3(u_i{}^2 + u_j{}^2)^{1/2}]\}/2$$

The control procedure may now be summarized as follows: 1) Sense the pitch, toll, and yaw angles and rates, orbital position and the apparent position of the sun. 2) Compute the control moments demanded by the system, using Eqs. (9). 3) Evaluate the maximum attainable moments using Eqs. (10). 4) Compare the moment demand with the attainable values. If the demand exceeds the maximum available, set it equal to the latter. 5) Obtain the required plate rotations δ_1, δ_2 and δ_3 through Eqs. (7).

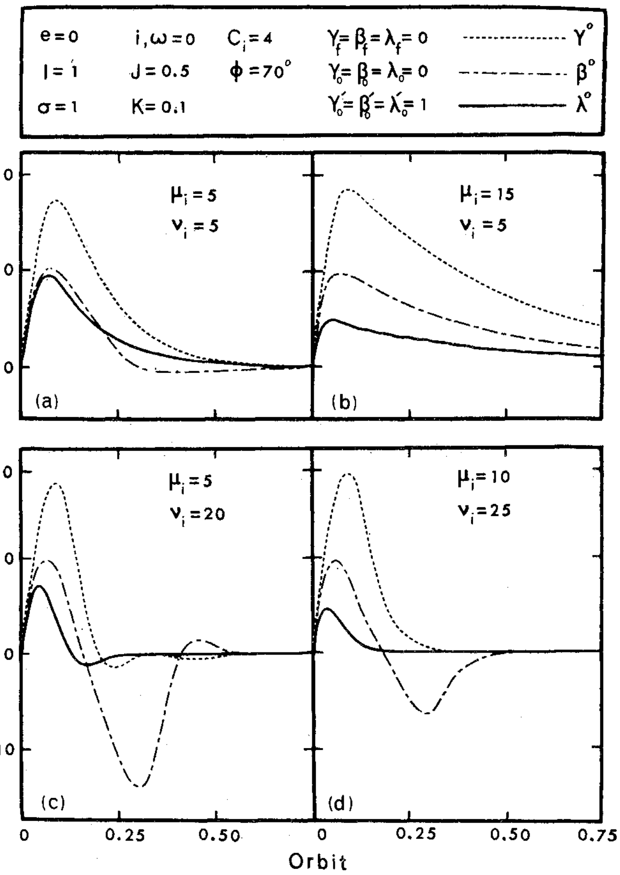

Fig. 4 Influence of controller gains μ_i, ν_i on the response.

Results and Discussion

The response of the system was studied by numerically integrating the equations of motion (3) along with the controller characteristic relations (9). The Adams-Bashforth predictor-corrector quadrature with the Runge-Kutta starter was used, in conjunction with a step size of 3°, which gave results of sufficient accuracy without involving excessive computational effort. The important system parameters were varied gradually over the range of interest and the controller performance evaluated both in circular and elliptic orbits (Table 1). The amount of information thus generated is rather extensive; however, for conciseness, only the typical results sufficient to establish trends are presented here. In general, the system is exposed to extremely severe disturbances, much higher than it is likely to encounter in the normal operation, to evaluate the controller's performance under adverse conditions.

Nutation Damping

Figure 4 summarizes the influence of the controller gains μ_i ν_i on the response. In general, an increase in μ_i provides an overdamped character to the system (Fig. 4a and b) while a corresponding increase in ν_i for a given μ_i results in oscillations suggesting a reduction in damping (Figs. 4a and c). The existence of an optimum choice of controller gains for a set of given system parameters is thus apparent. This is indicated in Fig. 4d.

The effects of the satellite inertia parameter I and the spin parameter σ, presented in Fig. 5, suggest that short, disklike satellites ($I = 1.5$) withstand and damp a given disturbance relatively better than long, slender satellites ($I = 0.5$). It is of interest to point out that here the value of J, representing the ratio of the axial inertias of the platform and the satellite, is taken as 0.5. The analyses with J varying from 0.25 to 0.75 showed the system response to remain virtually unaffected. This is understandable as large J makes it more amenable to controller corrections. The effect of the spin parameter σ was also found to be relatively insignificant. This is to be expected as the natural stiffness of the system, to which the spin parameter contributes, is largely provided by the controller gain ν_i (Figs. 5c and d).

Figure 6 shows the influence of the solar parameters C_i and aspect angle ϕ on the controller performance. The limitation imposed by C_i on the maximum allowable control moments

Table 1 Important system parameters

Parameter	C	I	J	e	i	μ_i, ν_i	σ	ϕ
Description	solar parameter	inertia parameter	platform inertia fraction	eccentricity	orbital inclination	system gains	spin parameter	solar aspect angle
Range varied	0–10	0–2	0.25–0.75	0–0.4	0°–90°	0–30	0–5	0–360°

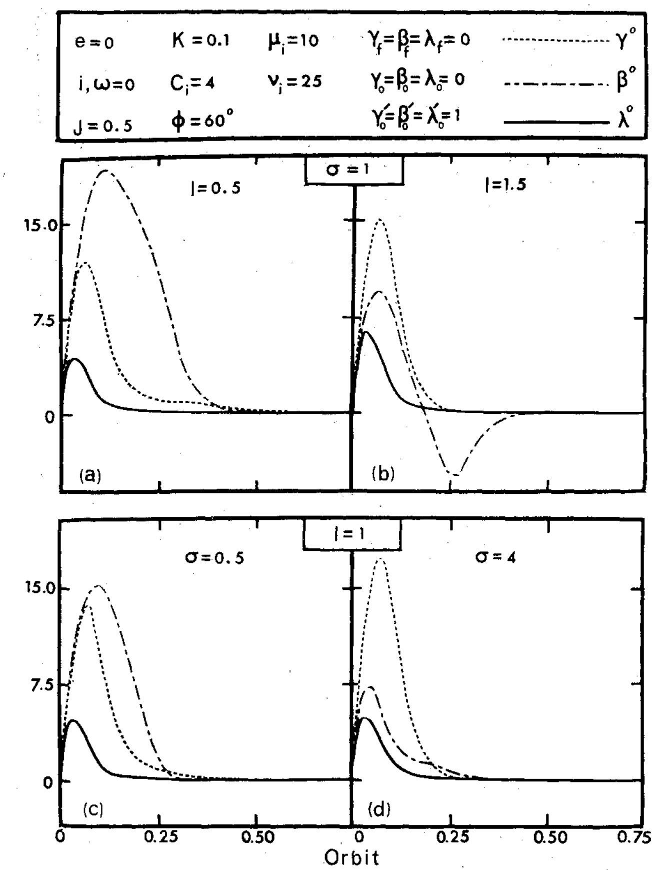

Fig. 5 Damped response as affected by satellite inertia and spin parameters.

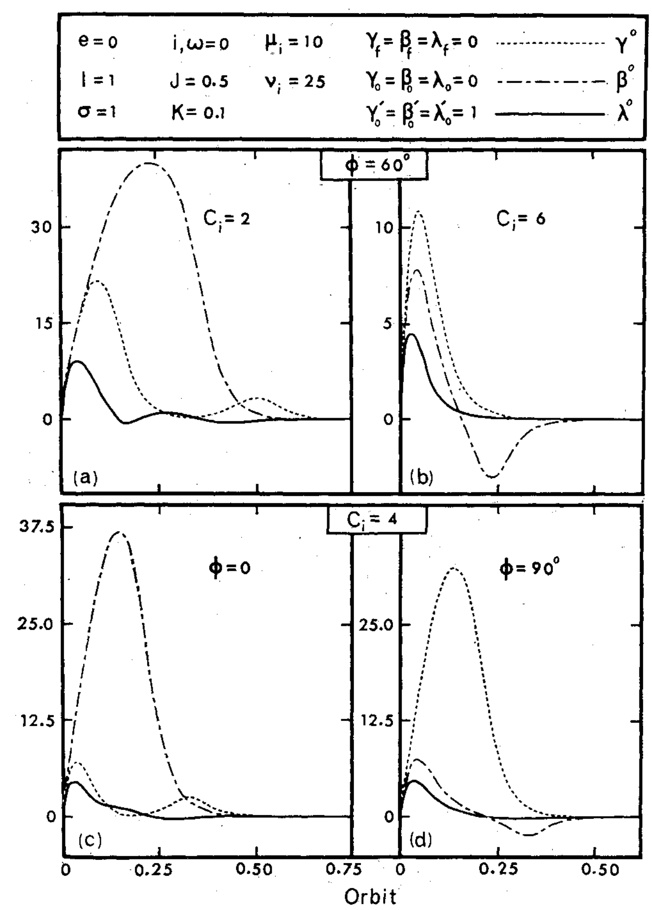

Fig. 6 Effect of solar parameters and solar aspect angle on the damped response.

is clearly reflected in the response plots (Figs. 6a and b). Note that the increase in C_i results in an over-all improvement in the transient performance of the system with the reduction in nutational amplitudes and the damping time. However, there is a restriction as to the maximum attainable values for C_i as imposed by the control plate areas and their moment arms.

The relative position of the sun (ϕ) affects the response in the γ and β degrees of freedom through the corresponding change in the available control moment. Thus roll and pitch attain different relative amplitudes in an orbit; however, their damping time remains essentially unaffected (Figs. 6c and d).

The performance of the controller in an eccentric orbit remains essentially the same except for a steady-state limit cycle appearing in the λ degree of freedom due to the periodic forcing function dependent on e (Eq. 3c). This is indicated in Figs. 7a and b, where roll and pitch motions damp out quite quickly but the platform yaw persists as a limit cycle. The sustained oscillation of the platform would be, in general, highly undesirable. Fortunately, it can be eliminated using a modified control relation for Q_λ sensitive to the eccentricity induced disturbance

$$\lambda_c = (1/\nu_\lambda)[-2e\sin\theta/(1+e\cos\theta) + \{(K/JI)(1+e)^{3/2}/ \times (1+e\cos\theta)^2\}\{1-(\sigma+1)(1+e)^2/(1+e\cos\theta)^2\}] \quad (9c)'$$

Figures 7c and d illustrate the effectiveness of the modified control relation in eliminating the limit cycle.

The information presented so far pertains to the orbital motion in the plane of the ecliptic ($i = 0$). Of course, depending upon the mission, the orbital plane would be at an angle to the ecliptic. A systematic study showed the effect of i to be confined to local changes in response character without significantly altering the overall control performance. The plots in Figs. 8a and b substantiate this conclusion.

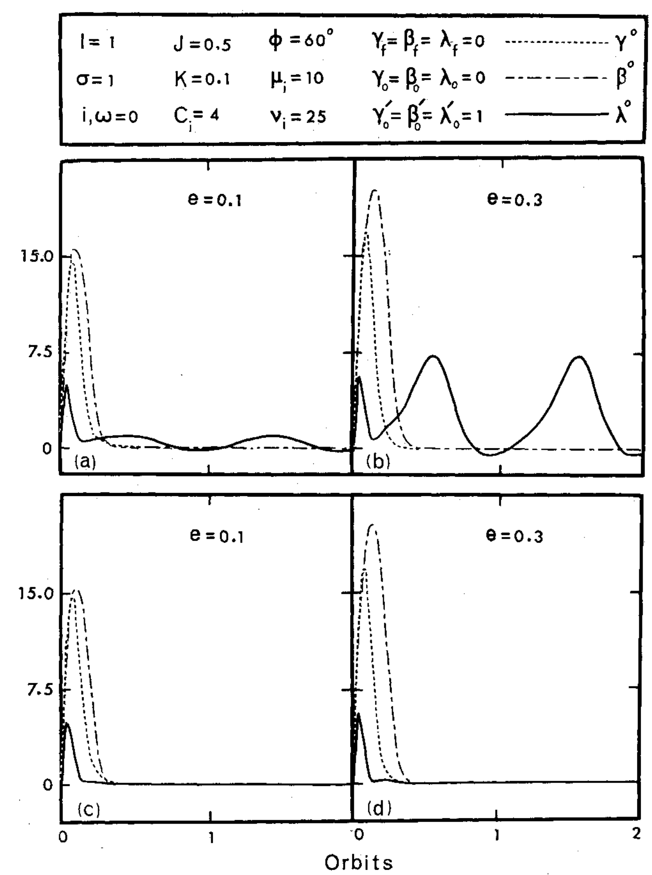

Fig. 7 Response plot showing limit cycle oscillations in eccentric orbits and their removal through the modified control function.

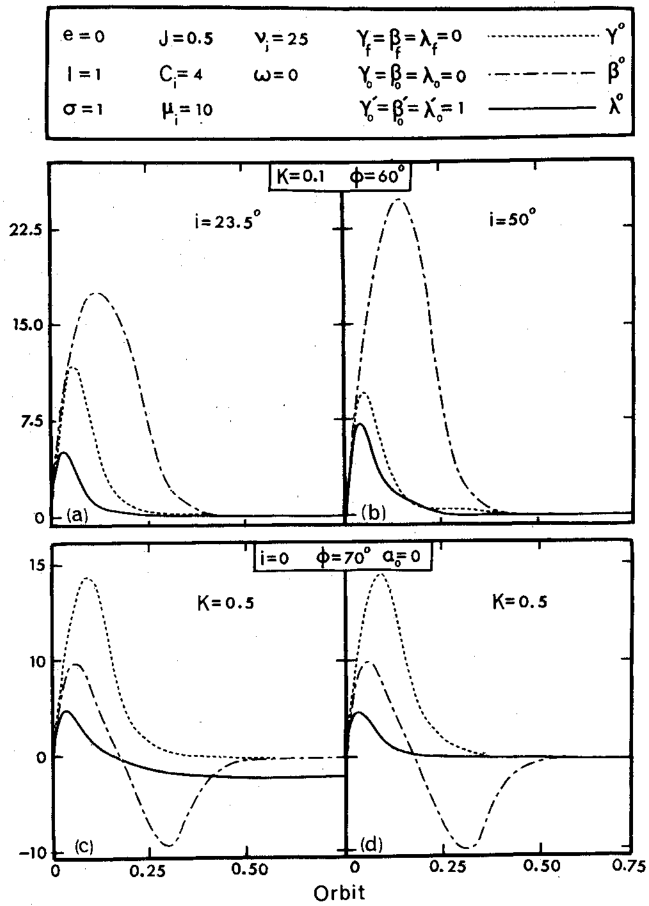

Fig. 8 Response as affected by: (a, b) orbital inclination from the ecliptic; (c) rotor spin decay; (d) modified control function accounting for dissipation.

Rotor Spin Decay

The present analysis considers the rotor (body I) to have a constant average spin rate. Apparently, this would be achieved through some active energy source. However, the influence of the possible rotor spin decay on the librational response of the system would be appropriate to explore. Although α still remains a cyclic coordinate, and, hence, the first integral is available, it is not possible to eliminate the rotor spin degree of freedom from the rest of the equations of motion. Thus one is faced with the solution of a seventh-order system (as against the sixth order in the previous case). As can be anticipated, the roll and pitch motions damp out as before, coupling with the α degree of freedom being weak. However, the platform yaw angle λ tends to drift away from its preferred orientation λ_f (Fig. 8c).

This does not reflect in any way a limitation on the capability of the controller, but failure on our part to exploit it fully. The validity of this observation becomes quite apparent when one examines the modified yaw equation in the presence of dissipation

$$\lambda'' - \gamma'' \sin\beta - \{2e \sin\theta/(1 + e \cos\theta)\}(\lambda' - \gamma' \sin\beta +$$
$$\cos\beta \cos\gamma) - \beta'\gamma' \cos\beta - \gamma' \cos\beta \sin\gamma - \beta' \cos\gamma \sin\beta +$$
$$(K/JI)\{(1 + e)^{3/2}/(1 + e \cos\theta)^2\}(\lambda' - \alpha') = Q_\lambda \quad (3c)'$$

Note the dynamic coupling between the rotor and the platform. On the other hand, the control relation for Q_λ (Eq. 9c) does not involve α' explicitly. Thus the controller's potential to account for the rotor spin decay is not utilized. The situation can easily be corrected by modifying the control function as indicated,

$$Q_\lambda = -\mu_\lambda\lambda' - a(\theta)\alpha' - \nu_\lambda(\lambda - \lambda_c) \quad (9c)''$$

where

$$a(\theta) = (K/JI)(1 + e)^{3/2}/(1 + e \cos\theta)^2$$
$$\lambda_c = -(1/\nu_\lambda)(2e \sin\theta)/(1 + e \cos\theta)$$

A typical system response using this modified control function is presented in Fig. 8d.

Implication of this analysis is rather far reaching. It is no longer necessary to maintain the rotor spin rate from attitude dynamics considerations as the controller is able to provide sufficient torque irrespective of the spin rate. The system thus has a truly semipassive character promising an increased satellite lifespan.

Attitude Control

The controller provides an interesting possibility of changing the satellite's preferred orientation in orbit. This is accomplished by using the position control parameters γ_c, β_c and λ_c in accordance with the desired equilibrium configuration γ_f, β_f, and λ_f. As an illustration, for a static equilibrium of the satellite in a circular orbit, Eqs. (3) in conjunction with Eq. (9) lead to

$$\gamma_c = (1/\nu_\gamma)[(\sin\gamma_f/\cos\beta_f)\{(1 - J)I(\sigma + 1) +$$
$$JI \cos\beta_f \cos\gamma_f\} + (3I - 4) \sin\gamma_f \cos\gamma_f] + \gamma_f$$
$$\beta_c = (1/\nu_\beta)[\cos\gamma_f \sin\beta_f\{(1 - J)I(\sigma + 1) +$$
$$(JI - 1)\cos\beta_f \cos\gamma_f\} - 3(I - 1) \sin^2\gamma_f \sin\beta_f \cos\beta_f] + \beta_f$$
$$\lambda_c = (1/\nu_\lambda)[(K/JI)\{\cos\beta_f \cos\gamma_f - \sigma - 1\}] + \lambda_f \quad (11)$$

Note that in this particular case of $e = 0$, the position control parameters are fixed once the final orientation is specified. On the other hand, for the case of an eccentric orbit, the parameters depend, in addition, on the satellite position in the orbit. Figure 9 illustrates the versatility of the semipassive

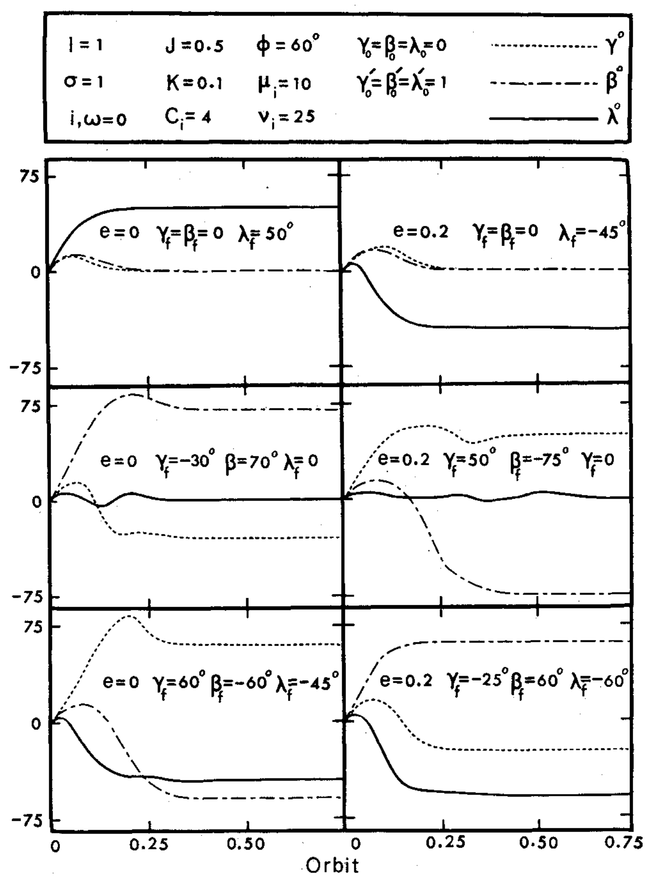

Fig. 9 Effectiveness of the controller in achieving arbitrary orientations of the body II.

controller in achieving arbitrary orientations in space for both circular and elliptic orbits. This suggests an exciting possibility for a space vehicle to extend its range of applicability and undertake diverse missions.

Illustrative Example

It was decided to demonstrate the effectiveness of the concept through a preliminary attitude dynamics study of the two well-known satellites—INTELSAT IV and Anik-1—when provided with the proposed controller. Appropriate geometrical and inertia properties were assigned. As seen before it is no longer necessary to spin a satellite for attitude control. However, nominal value of $\sigma = 1$ was taken to account for possible spin introduced from other considerations, e. g., temperature control. A plate area of 3 ft^2 and the moment arms $\varepsilon_i = 5$ ft yield the solar parameters $C_i = 2$ for INTELSAT IV and 4.5 for Anik-1. Subjecting the satellites to a disturbance equivalent to that imparted by micrometeorite impacts over 24 hr, which represents an enormous magnification of the real situation, gives[8] $-0.05 \leq \gamma_0', \beta_0', \lambda_0' \leq 0.05$. It is apparent (Fig. 10) that the controller is able to damp such a severe disturbance quite effectively with the maximum deviation from the preferred orientation of less than 0.25°. The figure also shows the controller's effectiveness in achieving specified spatial orientations. The semipassive character of the system limits the peak power requirement to $\simeq$ 5 w.

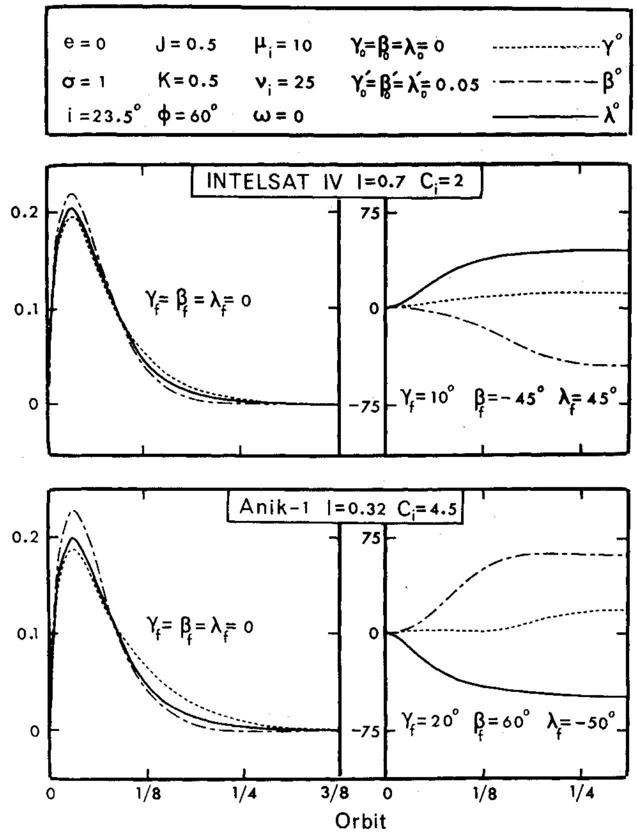

Fig. 10 Projected controller performance in achieving nutation damping and attitude control of INTELSAT IV and Anik-1 satellites.

Finally, a comment concerning the Earth shadow, which would render the controller ineffective, is appropriate here. For a geostationary orbit, the influence of shadow is confined to a quarter of the satellite's lifespan, and, even here, only during 5% of the orbital period. The results showed the controller performance to remain virtually unaffected. Furthermore, it should be pointed out that the analysis ignores dynamics due to relative motion of the control surfaces and to their shadowing by the satellite.

Conclusions

The significant conclusions based on the analysis may be summarized as follows: 1) The feasibility of a semipassive controller using solar radiation pressure for the nutation damping and attitude control of spinning satellites is clearly demonstrated. 2) The system is capable of damping extremely severe disturbances in a fraction of an orbit. The time of damping can be further reduced by an optimum choice of the system gains μ_i, ν_i. 3) It is possible for a satellite to attain any arbitrary orientation in space, both in circular and elliptical orbits. The controller thus imparts versatility to a space vehicle in undertaking diverse missions. A directional device aboard the stabilized platform may be Earth-oriented, space-oriented, or made to track a specified celestial object through a proper choice of the position control parameters. 4) The effectiveness of the controller remains unaffected, even during a spin decay, with a proper choice of the modified control function. Thus, it is no longer necessary to maintain a constant spin rate by compensating for the dissipated energy. 5) The semipassive character of the controller promises an increased satellite lifespan.

References

[1] Modi, V. J. and Flanagan, R. C., "Effect of Environmental Forces on the Attitude Dynamics of Gravity Oriented Satellites: Part I-High Altitude Orbits," *Aeronautical Journal*, Vol. 75. Nov. 1971, pp. 783–793

[2] Modi, V. J. and Flanagan, R. C., "Librational Damping of a Gravity Oriented System using Solar Radiation Pressure," *Aeronautical Journal*, Vol. 75, Aug. 1971, pp. 560–564.

[3] Modi, V. J. and Tschann, C., "On the Attitude and Librational Control of a Satellite using Solar Radiation Pressure," *Proceedings of the XXI International Astronautical Congress*, edited by L. G. Napolitano, North-Holland Publishing Co., Amsterdam, The Netherlands, 1971, pp. 84–100.

[4] Modi, V. J. and Kumar., "Librational Dynamics of Gravity Oriented Satellites under the Influence of Solar Radiation Pressure," *Proccedings of the International Symposium on Computer-Aided Engineering*, edited by G. M. L. Gladwell, University of Waterloo Press, Waterloo, Ontario, Canada, May 1971.

[5] Modi, V. J. and Kumar, K., "Coupled Librational Dynamics and Attitude Control of Satellites in Presence of Solar Radiation Pressure," *Proceedings of the XXII Congress of the International Astronautical Federation*, Brussels, Belgium, Sept, 1971; to be published.

[6] Modi, V. J. and Pande, K. C., "Solar Pressure Control of a Spinning Satellite with a Stabilized Platform," Mechanical Engineering Rept. ME 72-24, Dec. 1972, Univ. of British Columbia, Vancouver, Canada.

[7] Scull, J. R., "Mariner IV Revisited, or the Tale of the Ancient Mariner," presented at the *20th Congress of the International Astronautical Federation*, Argentina, Oct. 1969.

[8] Ehricke, K. A., *Principles of Guided Missile Design*, Van Nostrand, Princetown, N. J., 1960, pp. 241–256